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Acoustic gaps in a chain of magnetic spheres

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Acoustic gaps are normally observed in granular inhomogeneous structures made of composite materials. The modulation of the elastic properties in such media creates the coherent effects of scattering and interference that ultimately lead to frequency intervals where sound propagation is forbidden. Contrastingly, we report here an experimental observation of acoustic gaps in homogeneous media; specifically, in granular chains. The beads used in our study are magnetic. Therefore, instead of modulating the elastic properties of the chain, we modulate the magnetization (i.e., the contact forces). We also observe that the propagation speed of acoustic signals through the magnetic chains used in this study is at odds with the speed predicted by Hertz's law.

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I. INTRODUCTION

The propagation of acoustic signals through a chain of identical spheres has achieved, over the years, the status of a key model to study the sound properties of granular materials. Compressed [1–3] and uncompressed [1,4,5] chains have been studied with the aim of elucidating the important differences between the linear and nonlinear responses observed in granular systems. In principle, two adjacent beads in a compressed chain deform according to Hertz law: the deformation δ_0 being proportional to $F_0^{2/3}$, where F_0 is the contact force. If the amplitude of the oscillations is much smaller than δ_0, a well-known result for a chain of identical point masses linked by linear springs is recovered [1], where the spring constant k is proportional to $F_0^{1/3}$. Accordingly, since the sound velocity in the chain is proportional to $k^{1/2}$, this in turn is proportional to $F_0^{1/6}$. As a consequence, when F_0 is zero, the chain is unable to transmit sound. Nesterenko and co-workers found, however, that solitary waves can propagate through uncompressed chains if the amplitude of the oscillations is much larger than δ_0 [4–6]. The knowledge gathered so far about this nonlinear case has induced this group to recently propose novel applications, including trapping and shock disintegration [7] or bifurcating devices [8].

In this paper, we revisit the phenomenon of sound transmission through a chain of spherical beads. However, the spheres used in our study are magnetic. Magnetism provides cohesion, therefore, a chain of such beads can be formed without the need of an external load [see Fig. 1(a)]. Our aim here is to discuss two results. First, the speed of an acoustic signal through a magnetic chain follows a power law with the magnetic contact force F_m, but the exponent we found is 1/3 instead of the 1/6 obtained in compressed chains. Second, frequency gaps are obtained by modulating not the elastic constants of the spheres, but their magnetization. In order to properly asses the importance of this intriguing behavior, let us emphasize that acoustic band-gap effects have been observed only in a periodic composite consisting of two different materials [9,10]. The larger the elastic contrast between the constituents of the array, the better defined is the frequency gap. For instance, an externally loaded chain made of steel spheres, with Nylon spheres periodically distributed inside, might have a sound transmission spectrum with at least one band gap. In the strongly nonlinear case, where solitons are routinely observed, a granular composite of this sort is also able to confine and disintegrate solitary waves [7,11].

II. MAGNETIC INTERACTION

The interaction energy between two dipole moments $\mu_\mathbf{i}$ and $\mu_\mathbf{j}$ is given by $U_{ij}=\mu^2 r_{ij}^{-3}[\hat{\mu}_i\cdot\hat{\mu}_j-3(\hat{\mu}_i\cdot\hat{r}_{ij})(\hat{\mu}_j\cdot\hat{r}_{ij})]$, where $r_{ij}=|\mathbf{r}_j-\mathbf{r}_i|$ is the separation distance between the dipoles and $\mu=|\mu_i|=|\mu_j|$. Although this expression is only valid for ideal dipoles, we can use it to estimate the contact force between two adjacent spheres, $F_{i,i+1}$, in a chain of N magnetized particles. We plot in Fig. 2 the contact forces for two different dipole configurations: (a) the case where the

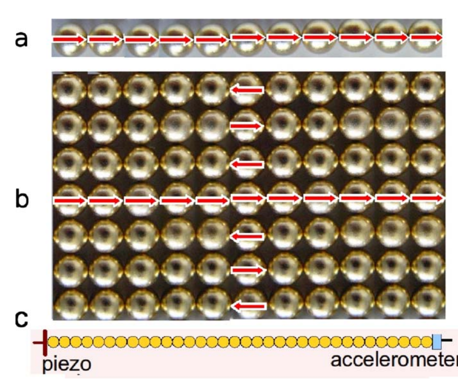

FIG. 1. (Color online) (a) A chain of magnetic spheres. The dipoles are aligned in a head-to-tail fashion (→→→→). (b) A two-dimensional lattice formed by magnetic spheres. Along one of the axis, the dipoles are aligned as shown in (a) and in the other they alternate orientations (left and right). (c) The experimental setup used in the experiments.

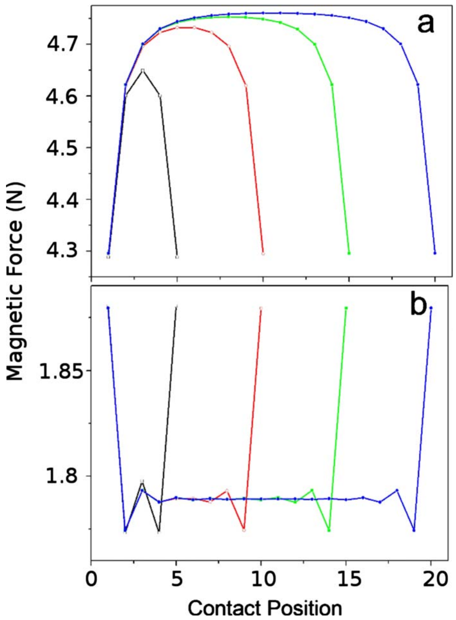

FIG. 2. (Color online) (a) Contact forces for chains of different lengths, with $N=6,11,16,21$. The configuration of the dipoles is in the head-to-tail fashion. Note that the contact forces are smaller at the boundaries of the chains due to the nature of the magnetic interaction. (b) Contact forces for chains with $N=6,11,16,21$, where the dipoles alternate orientations. Note that the contact forces are smaller (greater) at the boundaries of the chains due to the nature of the dipole-dipole interaction.

dipoles are oriented in a head-to-tail fashion ($\rightarrow\rightarrow\rightarrow\rightarrow\cdots$) and (b) the case where they alternate orientations. It is worth to remark that although both configurations are cohesive (see Fig. 2), only the first one is stable in real conditions. Indeed, any minor misalignment in the orientation of the dipoles in the second configuration produces a rapid sphere rotation and the complete chain flips to the head to tail form. Interestingly, however, this second array becomes stable if head to tail chains are assembled in a two-dimensional (2D) lattice [see the central chain in Fig. 1(b)] [12].

III. EXPERIMENTAL DETAILS AND RESULTS

We start measuring the sound speed in a chain of 20 magnetic spheres as a function of the magnetic contact force. The spheres have a diameter d of 5 mm and were purchased (SuperMagnetM) with a permanent magnetic dipole which we estimated to be $\mu=0.07$ A m^2 (this value corresponds to a surface field intensity of 6.4 kG, measured with a Lakeshore Gaussmeter 475). The spheres are made of a NdFeB mixture and are covered by the manufacturer with a hard epoxy layer. Using a temperature-controlled furnace, the beads are heated to reduce their magnetization. Therefore, chains can be formed with different cohesive forces, which are easily obtained by carefully measuring the force needed to separate the chains by the middle. A piezoelectric is put in contact with the first sphere of the chain and an accelerometer (DeltaTron-BK, 1000 mV/g) is attached to the last one [see Fig. 1(c)]. The piezoelectric, driven by a function generator (HP-33120A), injects to the chain a 2 μs square pulse. By measuring the time of flight of the front edge of this narrow pulse with an oscilloscope Agilent 54641A (see Fig. 3) and knowing the length of the chain (in this case, 10 cm), the speed of sound is obtained (Fig. 4). We found that the velocity measured in this way compares reasonably well to the velocity of a real wave, which can be produced as an steady-state wave at some frequency lower than a cutoff frequency. Since the speed of sound is proportional to the cutoff frequency (see [1]), the speed can be estimated if we experimentally find this frequency (see the red dots in Fig. 4). Our results indicate that the narrow pulse used in the first method does not spread much during the propagation.

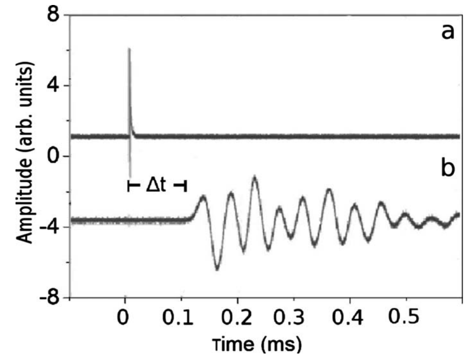

FIG. 3. (a) Acoustic pulse sent through a chain of magnetic spheres. The pulse has a width of 2 μs. (b) The signal measured at the end of the chain. Δt is the time of flight of the pulse in the chain. Note that the accelerometer takes around 400 μs to relax.

We observe that the greater is the contact force between the beads, the higher is the sound speed (see Fig. 4). The data are reasonably well fitted by a power law with an exponent of 1/3 (see also the log-log plot in the inset of the figure). In

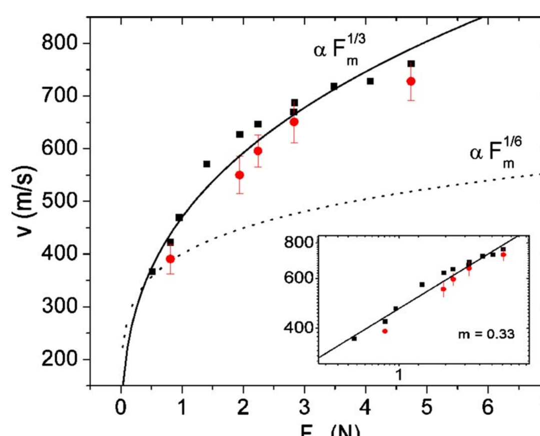

FIG. 4. (Color online) Speed of sound as a function of the contact force in the middle of the chain. Data are well fitted by a power law with an exponent 1/3 (see also the log-log plot in the inset). Black squares were measured as indicated in the text and red dots were obtained by first measuring the cutoff frequencies (frequencies at which no signal propagates) and then using the expression $V=2\pi a f_c$, where a is the radius of the beads (see [1]). We show, for comparison, the speed of sound predicted by Hertz's law (exponent 1/6).

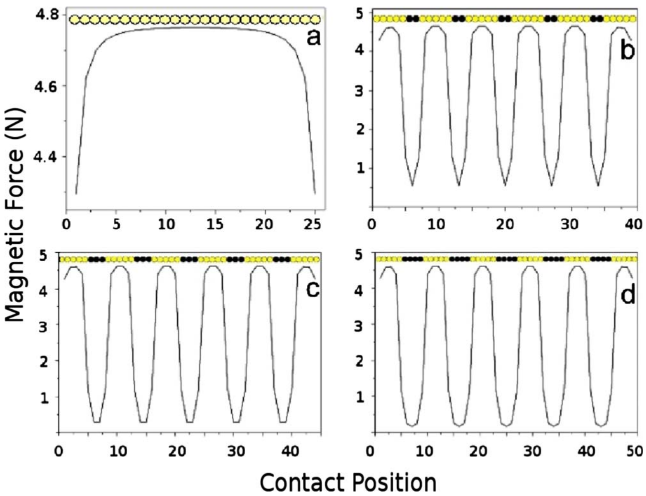

FIG. 5. (Color online) Contact forces in short chains as a function of length. Each chain is formed by five and a half cells and each cell by five magnetic (yellow) and (a) no, (b) two, (c) three, and (d) four nonmagnetic spheres (black).

principle, this value differs from the classical result given by Hertz model, which predicts an exponent of 1/6 [1] (see dotted line in Fig. 4). It is important to mention, however, that an exponent of 1/4 has been found in other experiments [3,13,14]. Although the nature of the 1/4 exponent is still an unsettled controversy in the literature, it is believed that this exponent may have its root in a modified Hertz's law that takes place when the compression force is significantly reduced, from around 200 to 20 N [3]. Hence, we speculate that the exponent we found in our experiment is greater than 1/4 because the contact forces are further reduced (to around 3 N, as seen in Fig. 4). More research is needed to elucidate the nature of this exponent.

As mentioned before, a second experiment was carried out to see if the magnetic chain behaves as a phononic crystal when nonmagnetic (i.e., fully demagnetized) beads are periodically inserted inside. In principle, this insertion could modulate the acoustic impedance within the chain, affecting the outcome of a sound wave traveling through it. The chains are formed by $N=25$ cells, each cell, in a given chain, having

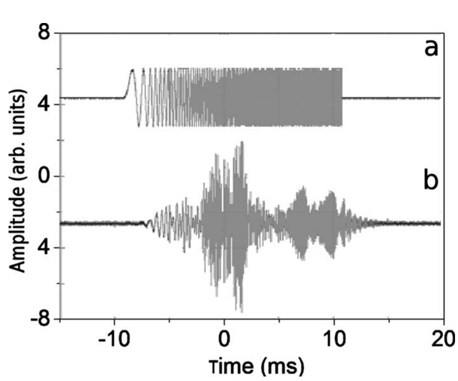

FIG. 6. (a) Acoustic signal sent through a chain of magnetic spheres. The signal is composed by a 20 ms wave train, where the frequency increases with time from 1 kHz to 15 kHz. (b) Signal measured at the end of the chain.

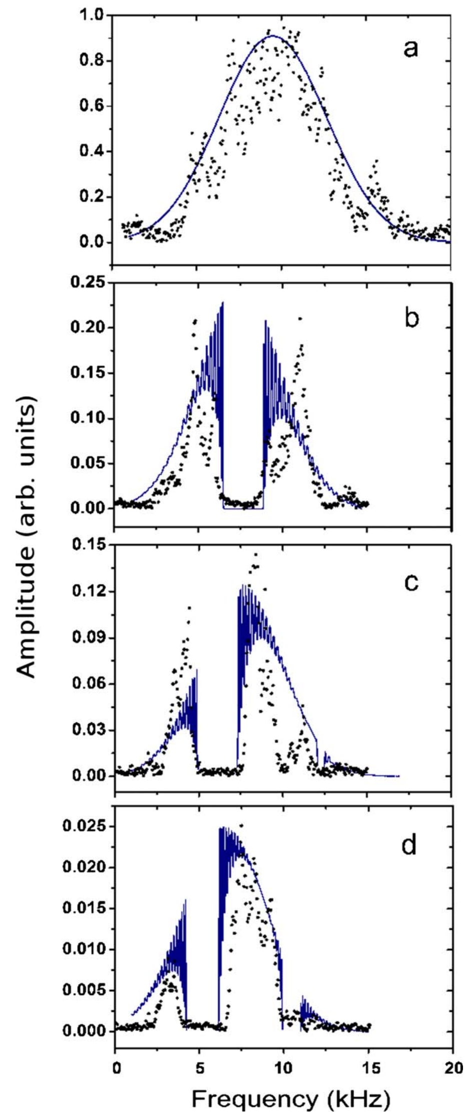

FIG. 7. (Color online) Power spectra of the transmitted pulse as the number of nonmagnetic spheres (n) is increased; (a) $n=0$, $\eta=1$, $f_0=9.5$ kHz, $\sigma=4.5$ kHz, and $\lambda=0.95$; (b) $n=2$, $\eta=0.61$, $f_0=7.6$ kHz, $\sigma=3.5$ kHz, and $\lambda=0.22$; (c) $n=3$, $\eta=0.58$, $f_0=6.1$ kHz, $\sigma=3.5$ kHz, and $\lambda=0.14$; (d) $n=4$, $\eta=0.54$, $f_0=5.2$ kHz, $\sigma=3.5$ kHz, and $\lambda=0.025$. Black dots are experimental data and the (blue) curve the theoretical prediction.

five magnetic and n nonmagnetic beads, where n can be 1, 2, 3, or 4. Once the nonmagnetic beads are placed inside the chain, they, of course, remagnetize a bit, guaranteeing the cohesion of the chain. However, the greater is n, the weaker the contact forces in such segments and thus, the higher the contrast between these and the magnetic beads. Figure 5 helps to illustrate the modulation of these contact forces in a chain made of five and a half cells.

In principle, there are two ways to obtain the power spectrum of a sonic pulse traveling through a linear chain: the so-called normal-mode analysis (NMA) and the pulse analysis (PA) (see, for example, [15]). In the first one, an oscillation with a given frequency is transmitted through the structure and as this frequency is varied, the arriving signal is recorded. In the second one, a pulse, rich in frequencies, is generated and sent through the chain. The signal is measured with a receptor and then Fourier transformed. The PA method is much faster, but the frequency composition of the pulse has to be previously determined. We implemented a third method, closely related to the second one. It consists in propagating a train of sinusoidal waves. It consists of a wave where the frequency increases in time. The detector acts as an integrator and the recorded signal is Fourier transformed. Figure 6 shows a typical train and a typical received signal.

We show in Fig. 7 the power spectra of the transmitted wave for the four chains considered. As previously stated, the chain are formed by 25 cells and each cell incorporates an increasing number of impurities (nonmagnetic spheres). There is clearly a frequency gap in the last three spectra. The case for $n=1$ is not shown because it is similar to the case where all the beads are magnetic $n=0$. We observe that the spectra, together with the gaps, shift to lower frequencies as n grows. Considering that the length of the chain grows, this shift is expected.

IV. DISCUSSION

A theoretical model to back-up our experimental results is presented next. The model uses a handy mathematical expression reported by Griffiths and Steinke [16], who solved the general theory of wave propagation in locally (finite) periodic media, using the framework of the transfer matrix. A sinusoidal signal of a given frequency f, propagating through a finite periodic one-dimensional (1D) system like the one shown in Fig. 5, is transmitted with a transmission coefficient given by [16]: $T=[1+z^2(\frac{\sin N\gamma}{\sin\gamma})^2]^{-1}$, where $z=\epsilon_-\sin(\kappa_2 b)$, $\gamma=\cos^{-1}(\xi)$, and $\xi=\cos(\kappa_2 b)\cos(\kappa_1 L)-\epsilon_+\sin(\kappa_2 b)\sin(\kappa_1 L)$. ϵ, κ, η, b, and L are defined as follows: $\epsilon_\pm=1/2(\eta\pm 1/\eta)$, $\kappa_i=(2\pi f)/v_i$, $\eta=\kappa_1/\kappa_2=v_2/v_1$, $b=nd$, and $L=5d$. Since all the beads in the chain (magnetized and nonmagnetized) are elastically identical, η is simply the ratio of the acoustic impedances and this ratio is taken as the only free parameter to fit the position and shape of the acoustic gaps. However, we have to solve first the following inconvenient: Griffiths and Steinke's theory restricts itself to the case of nondissipative waves, while in our case the sound propagation is strongly dissipative. Therefore, we propose to simply multiply T by an *ad hoc* function that gives us the possibility to mimic the attenuation and its frequency behavior. Then, the modified transmission coefficient T_m is $T_m=\lambda\exp[-\frac{(f-f_0)^2}{\sigma^2}]T$, where f_0 is the frequency at which a given spectrum is centered, σ its width, and λ an attenuation coefficient that is obtained by inspection. The *ad hoc* function is Gaussian because the spectrum associated to $n=0$ has clearly this form. The reasonable agreement we found with the experimental results (see the line in Fig. 7) is worth to be mentioned. First, a compact mathematical expression that has its origin in a quantum-scattering theory is able to fully describe a finite mechanical system. Second, to fit the data, we used only one free parameter, η, and this parameter is simply the ratio of two velocities.

V. CONCLUSIONS

We conclude that a linear chain of beads with no external compression is able to propagate sound if the beads attract each other by an internal magnetic force. We found that the speed of an acoustic signal is proportional to $F_m^{1/3}$. In addition, and most important, if the magnetic interaction between beads (i.e., the contact strength) is modulated by the insertion of fully demagnetized beads of the same material, frequency gaps appear.

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